C11 - Real time monitoring of wide area power systems

11.1. Overview

Running a load flow algorithm requires a large quantity of data. Besides the general data, regarding the system's topology and branch electrical parameters, all active and reactive power injections from PQ buses, all bus active powers and voltage magnitudes and the slack bus voltage magnitude and angle must be provided.

For system management and control, in scenarios where accurate real time data is required (accidental bottleneck and contingency analysis, for instance), obtaining and processing such amount of data is not always possible. In these cases, other analysis algorithms are required, which can use less input data, but, at the same time, compute the load flow variables with enough accuracy.

Such algorithms are the static state estimation (SSE) algorithms, which, based only on limited data, taken from key points in the system, and knowing the network's structure, branch electrical data and current operating configuration, can compute an approximation of the system's current state, expressed as bus voltages and branch load flows. The process is called state estimation.

A state estimation algorithm computes a set of state variables which usually are the bus voltage, in magnitude and angle, which are subsequently used to find the other variables of a load flow calculation: branch power flows, active power losses, transformer loadings etc.

SSE algorithm use as inputs values measured directly from the system, called measurements. This feature makes them the primary tool in assessing the system's state in real time, where load flow calculations are needed in intervals of minutes. The types of measurements used by classic SSE algorithms are:

- bus active and reactive powers;
- branch active and reactive power flows;
- bus voltage magnitudes.

These are complemented by pseudo-measurements, which are values taken not from real time measured data, but are known in advance, such as:

- the voltage magnitude set in a PV bus;
- the null active and reactive power injections form transfer buses;
- forecasted values.

Measurements can be affected by errors. Some error sources are:

- a lower precision class of the metering equipments;
- metering equipment malfunctions and incorrect telemetry reading from the switches in the system;
- errors generated by equivalent electrical models of the real elements;
- lack of synchronism between measurements.

The latter is the reason why classic SSE algorithms do not use as measurements bus voltage angles. On one hand, the SCADA/EMS systems provide values at a interval of seconds, too slow for the variation speed of the voltage sinusoidal wave. On the other hand, these measurements cannot be synchronized accurately enough, and using of phase measurements can lead to poorly estimated results.

11.2. The observability concept

A system is considered observable if, based on the given set of measurements, all its state variables can be computed. Otherwise, the system is called unobservable.

A system can be unobservable because of:

- an insufficient number of measurements;
- incorrect placement of measurements;
- use of a poor estimation algorithm.

A way of ensuring observability is to use a sufficient number of measurements (measurements redundancy), and one of the functions of the state estimation algorithm is to detect and remove bad measurements.

Two types of observability are defined:

- numeric observability, which describes the possibility of solving the equations system which defines the SSE model;
- topological observability, which describes the possibility to build a complete tree of measurements for the analyzed system. In this tree, all the buses from the system are connected to branches to which computed or measured current flows are associated.

The numeric observability guarantees the topological observability. The reverse statement is not necessarily true.

11.3. The general mathematical model of the state estimation

If m is the number of measurements

z_i – the measured value i;

h_i – non-linear function of variable x_i;

ε_i – the error of the measurement i;

then the state estimation model with the unknowns x_k, $k = 1...2n-1$ is written as:

$$z_i = h_i\left(x_1, x_2 ....., x_{2n-1}\right) + \varepsilon_i \qquad i = 1..m \tag{11.1}$$

or, in matrix form:

$$[z] = h([x]) + [e] \tag{11.2}$$

where:

$[z]$ – vector of measured values;

$[x]$ – vector of state variables;

$h([x])$ - a known non-linear function $h{:}R^n x R^m$ which relates measurements to state variables;

e – error vector [Alexandrescu 97].

The state estimation consists of computing the approximate values of the state variables (the magnitudes and angles of the bus voltages).

The complete set of modules of a state estimator is [Gavrilaş 08]:

- The network topology processing module, which reads the closed/open states of all circuit breakers and switches and gives the real working configuration of the analyzed system;
- The observability analyzer, which assesses if the given measurements set is sufficient for computing the system's state;
- The state estimator, which computes the best estimation of the state variables (bus voltage magnitudes and angles), using the available set of measurements and network data. The state variables are used subsequently to estimate the other load flow variables, such as the loading levels of the system elements;
- The bad data processing module, which finds the heavily erroneous measurements which could affect the precision of the estimated results.
- The system parameters and structure processing module, which estimates the general parameters of the system: line and compensation parameters, structural errors in the system's configuration.

The most known state estimation algorithms are:

- The Weighted-Least-Square (WLS) Algorithm [Schweppe 70.1]
- The Least absolute Value (LAV) algorithm [Abur 93], [Singh et.al. 97]
- The approximate state estimation model [Schweppe,70.2]
- The Levenberg-Marquardt algorithm [Rao 80]
- The Hachtel method [Gjelsvik et. al. 85]
- The orthogonal factorization method [Pajic 07]

11.4.The WLS Algorithm

The most known SSE algorithm is the weighted least squares (WLS) method, which computes the state variables so that the weighted sum of square deviations between the measured and the computed values for the measurements to be minimized. If the measurements set described by (11.1), $[z]$, is:

$$[z]=\begin{bmatrix} z_1 \\ z_2 \\ \dots \\ z_m \end{bmatrix}=\begin{bmatrix} h_1(x_1,x_2,\dots,x_n) \\ h_2(x_1,x_2,\dots,x_n) \\ \dots \\ h_m(x_1,x_2,\dots,x_n) \end{bmatrix}+\begin{bmatrix} e_1 \\ e_2 \\ \dots \\ e_m \end{bmatrix}=h([x])+[e] \tag{11.3}$$

with

$$h^T=[h_1([x])\ h_2([x])\ \dots h_m([x])]$$

where:

$h_i([x])$ - a non-linear function which computes the measurement *i* based on the state variables vector $[x]$;

$x^T=[x_1\ x_2\ \dots\ x_n]$ is the state vector;

$e^T=[e_1\ e_2\ \dots\ e_m]$ vector of Gaussian random measurement errors, with normal distribution.

The WLS estimation seeks to minimize the goal function:

$$J([x])=\sum_{i=1}^{m}(z_i-h_i([x]))^2\cdot w_i=\big[[z]-h([x])\big]^T\cdot W\cdot\big[[z]-h([x])\big] \tag{11.4}$$

where W is a diagonal matrix where the diagonal elements are the w_i weights associated to each z_i measurement, to express the „confidence ” in the value of that measurement. If the weight is 0, then the measurement is not used or present. A high weight value signifies that the measurement is believed to be trustworthy.

The best estimation for the vector $[x]$ is obtained when the weights matrix W is the inverse measurement error covariance matrix, when the m measurements are considered as independent:

$$R=Cov([e])=E([e]\cdot[e]^T)=diag(\sigma_1^2,\sigma_2^2,\dots,\sigma_m^2) \tag{11.5}$$

where σ_i^2 is the error variance of the measurement *i* and $E([e])$ is the average value of errors $[e]$. (11.4) becomes:

$$J([x])=\sum_{i=1}^{m}(z_i-h_i([x]))^2/R_{ii}=\big([z]-h([x])\big)^T\cdot R^{-1}\cdot\big([z]-h([x])\big) \tag{11.6}$$

(11.6) is minimized through simultaneous linearization of its derivatives in point *[x]*:

$$g([x]) = \frac{\partial J([x])}{\partial [x]} = -2 \cdot H^T([x]) \cdot R^{-1} \cdot ([z] - h([x])) \tag{11.7}$$

where $H([x]) = \left[\frac{\partial h([x])}{\partial [x]} \right]$

(11.7) is solved in an iterative process, by applying successive corrections $[\Delta x]^k$ to the current approximation $[x]^k$ of the solution:

$$[x] = [x]^k + [\Delta x]^k \tag{11.8}$$

If the current approximation is near the solution, i.e. the Δx_i corrections are small enough, the Taylor expansions of the measurement function near the current approximation, discarding its non-linear part, can be written:

$$h([x]) = h([x]^k + [\Delta x]^k) \approx h([x]^k) + H([x]^k) \cdot [\Delta x]^k \tag{11.9}$$

where $H([x]^k)$ is the Jacobian matrix of the measurement functions, computed in the same way as for the Newton-Raphson method.

With this approximation, the deviation between the $[z]$ measurements and the $h([x])$ computed values is:

$$[z] - h([x]) = [z] - h([x]^k) - H([x]^k) \cdot [\Delta x]^k \tag{11.10}$$

If the deviation of the computed values from the measurements is denoted $[z] - h([\mathrm{x}]^k) \overset{\text{not}}{=} [\Delta \mathrm{z}]^k$, (11.7) is rewritten as:

$$H^T([x]^k) \cdot R^{-1} \cdot \left([\Delta \mathrm{z}]^k - H([x]^k) \cdot \Delta[x]^k\right) = 0 \tag{11.11}$$

or

$$H^T([x]^k) \cdot R^{-1} \cdot H([x]^k) \cdot [\Delta x]^k = H^T([x]^k) \cdot R^{-1} \cdot [\Delta \mathrm{z}]^k \tag{11.12}$$

If $H^T([x]^k) \cdot R^{-1} \cdot H([x]^k) \overset{not}{=} G([x]^k)$ is the gain matrix for the $[x]^k$ approximation, (11.12) is written as:

$$G([x]^k) \cdot [\Delta x]^k = H^T([x]^k) \cdot R^{-1} \cdot [\Delta z]^k \tag{11.13}$$

(11.13) will give the $[\Delta x]^k$ corrections, which, applied to the $[x]^k$ current approximation, will lead to a new approximation of the solution:

$$[x]^{k+1} = [x]^k + [\Delta x]^k \tag{11.14}$$

The iterative process described by these expressions is repeated until a stopping criterion is met, for instance $Max\left\|[\Delta x]^k\right\| \le \varepsilon$.

11.5 Example

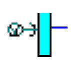
State estimation with the WLS algorithm on the IEEE 14 bus test system

This example demonstrates several scenarios in which the WLS state estimator is applied to the IEEE 14 bus system [web pstca], modeled in the Power Education Toolbox software [web PET].

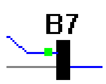
The estimator uses precise values for the measurements, taken from a load flow calculation.

Measurements are represented as follows:

- active / reactive bus power injection measurement



- bus voltage magnitude measurement



- branch active/ reactive power flow measurement

A. The unobservable system

In Fig. 11.1, there are not enough measurements to ensure the system's observability. The branches coloured in red are in the unobservable area. The problem is solved by adding measurements in the unobservable area.

B. Observable system, precise measurements

After the system is made observable, the WLS algorithm computes the state variables, bus voltage magnitudes and angle (Fig. 11.2). A quick comparison with the load flow results shows that, for the given number and placement of measurements, the results, while not being exact, are accurate enough (Case A). Using more measurements, the precision of the estimation improves and the algorithm gives better results, compared to the exact load flow estimation. The measurement list and estimation results are presented in Tables 11.1 and 11.2.

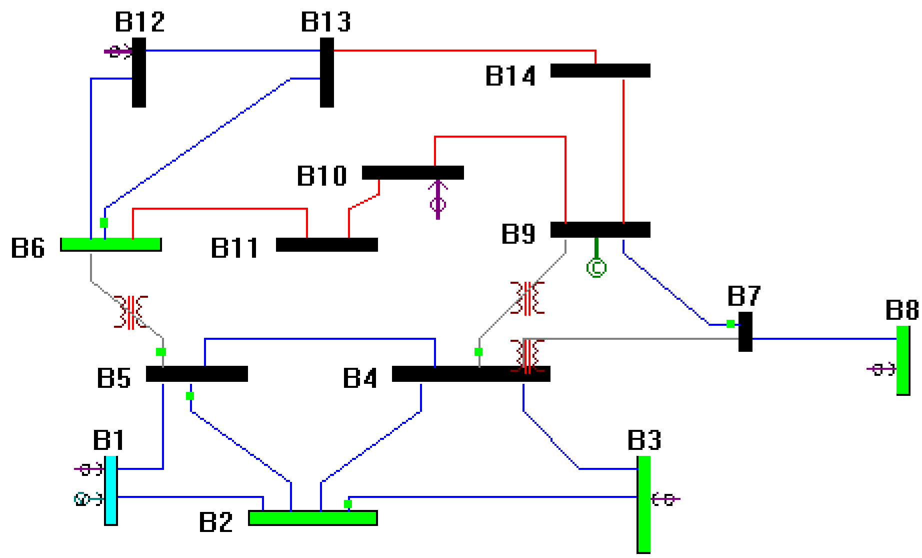


Fig. 11.1- An unobservable system

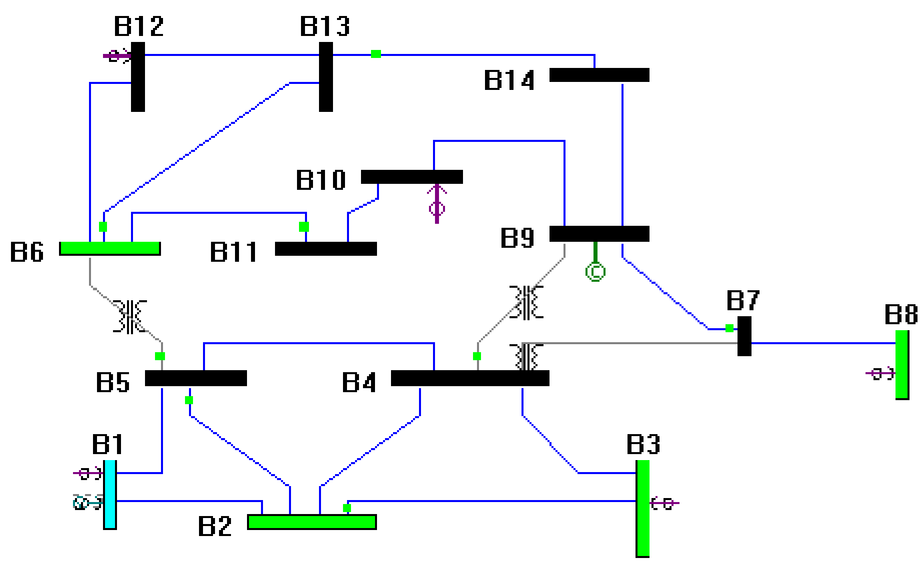


Fig. 11.2 - An observable system

Table 11.1 – WLS estimation results vs. load flow results

Bus	load flow			WLS estimation, low number of measurements		WLS estimation, high number of measurements	
	Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]		Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]	Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]
1	1.06	0		1.061	0	1.06	0
2	1.045	-4.98		1.046	-4.97	1.045	-4.98
3	1.01	-12.72		1.01	-12.71	1.01	-12.71
4	1.019	-10.32		1.019	-10.32	1.019	-10.32
5	1.02	-8.78		1.021	-8.78	1.02	-8.78
6	1.07	-14.22		1.071	-14.21	1.07	-14.22
7	1.062	-13.37		1.062	-13.38	1.06	-13.37
8	1.09	-13.37		1.09	-13.37	1.09	-13.37
9	1.056	-14.95		1.056	-14.96	1.056	-14.95
10	1.051	-15.10		1.051	-15.11	1.051	-15.11
11	1.057	-14.79		1.058	-14.79	1.057	-14.79
12	1.055	-15.08		1.056	1.051	1.055	-15.08
13	1.051	-15.16		1.051	-15.15	1.05	-15.16
14	1.036	-16.04		1.036	-16.04	1.036	-16.05

Table 11.2 - Measurement list for the IEEE 14 bus system

	Case A – low number of measurements	Case B – high number of measurements
Bus voltages	B1	B1
Bus powers	B1, B3, B8, B10, B12	B1, B2, B3, B7, B8, B10, B11, B12
Branch power flows:	B2-B3, B5-B2, B5-B6, B4-B9, B7-B9, B11-B6, B6-B13, B13-B14	B1-B2, B2-B3, B3-B4, B4-B2, B5-B2, B5-B4, B5-B6, B4-B9, B7-B9, B7-B8, B11-B6, B6-B13, B12-B13, B13-B14

C. Influence of bad measurements and measurement redundancy

For both cases, with low and high number of measurements, a malfunction of the measuring device for the branch power flow B2-B3 is considered. Instead of providing the actual flow measurement of P=73.18 MW and Q=3,56 MVAr, the measured values will be 0 and 0, and these values will be used by the WLS algorithm. In the first case, where the redundancy of measurements is low, reducing the weight of the bad measurement does not restore the precision of the algorithm and bad measurements affects heavily the results mostly in the area surrounding the bad measurement (Table 11.3).

Table 11.3 – WLS estimation results with bad measurement, the case with low number of measurements

Bus	WLS estimation			WLS estimation, high measurement error/ high weight in bus 3		WLS estimation, high measurement error/ low weight in bus 3	
	Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]		Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]	Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]
1	1.061	0		1.06	0	1.06	0
2	1.046	-4.97		1.045	-4.98	1.046	-4.97
3	1.01	-12.71		1.04	-4.92	0.892	-10.11
4	1.019	-10.32		1.095	-3.58	0.812	-1.53
5	1.021	-8.78		1.02	-8.79	1.021	-8.78
6	1.071	-14.21		1.07	3.58	1.071	-14.21
7	1.062	-13.38		1.14	0.94	0.854	-6.34
8	1.09	-13.37		1.166	0.95	0.888	-6.34
9	1.056	-14.96		1.134	-0.43	0.847	-8.79
10	1.051	-15.11		1.101	-4.78	0.904	-11.04
11	1.058	-14.79		1.057	-14.82	1.058	-14.79
12	1.056	-15.05		1.055	-15.09	1.056	-15.07
13	1.051	-15.15		1.05	-15.18	1.051	-15.15
14	1.036	-16.04		1.035	-16.07	1.036	-16.04

Table 11.4 – WLS estimation results with bad measurement, the case with high number of measurements

Bus	WLS estimation			WLS estimation, high measurement error/ high weight in bus 3		WLS estimation, high measurement error/ low weight in bus 3	
	Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]		Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]	Bus voltage magnitude [p.u.]	Bus voltage angle [degrees]
1	1.06	0		1.053	0	1.061	0
2	1.045	-4.98		1.039	-4.94	1.046	-4.97
3	1.01	-12.71		1.009	-11.57	1.011	-12.7
4	1.019	-10.32		1.013	-10.14	1.019	-10.31
5	1.02	-8.78		1.014	-8.67	1.021	-8.77
6	1.07	-14.22		1.064	-14.17	1.071	-14.21
7	1.06	-13.37		1.056	-13.24	1.062	-13.35
8	1.09	-13.37		1.084	-13.24	1.09	-13.35
9	1.056	-14.95		1.05	-14.84	1.057	-14.93
10	1.051	-15.11		1.045	-15.01	1.052	-15.09
11	1.057	-14.79		1.051	-14.73	1.058	-14.78
12	1.055	-15.08		1.049	-15.03	1.056	-15.07
13	1.05	-15.16		1.044	-15.12	1.051	-15.15
14	1.036	-16.05		1.029	-15.98	1.036	-16.02

In the second case, where the redundancy of measurements is high reducing the weight of the bad measurement does restore the precision of the algorithm. The use of bad measurement affects the results for the entire system, but, because of the presence of more good measurements, its effect is not so heavy as in the first case. Reducing the weight of the bas measurement restores the precision of the algorithm (Table 11.4).

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