C08 - The Newton-Raphson method

8.1. Review
In the matrix equation

$$[\underline{Y}_n]\cdot[\underline{U}_n]=[\underline{J}_n] \qquad (8.1)$$

where:

$[\underline{Y}_n]$ – the bus admittance matrix
$[\underline{U}_n]$ – column vector of the complex voltages from the system's buses
$[\underline{J}_n]$ – column vector of the complex current injections (loads) at the system's buses.
n – number of independent buses in the system

for solving the load flow problem, if all the bus current injections are known (the load pattern is known) and the bus admittance is computed (i.e. the physical structure of the system, which means connectivity, type and electrical parameters for all the branches, is fully known), the bus voltages can be computed directly, using the inverse of the $[\underline{Y}_n]$ matrix. In this case, the equation system described by (8.1) is linear and direct load flow methods can be applied.

However, in most cases, and especially for meshed systems, the bus loads are given as active and reactive powers, which are in a non-linear relation with the bus voltages. The load flow model thus becomes non-linear and it is solved applying iterative methods.

Regardless of the applied method, a load flow analysis consists of several steps [Alexandrescu 97]:

Input data acquisition
The input data can be summarized into three categories:

General data: *number of independent buses in the system, number of branches in the system, the location of the slack bus, the computing precision sought.*

Bus data: *the type of each bus in the system, with its known data.*

In the one-line diagram, the common ground connection of the system is chosen as reference bus, while all the other buses are considered as independent buses, and they are described using four basic parameters: the active power P; the reactive power Q; the voltage amplitude U and voltage angle θ.

In load flow calculations, two of these values are known, while the other two are to be computed. Based on this criterion, a bus can be:

- A PQ type (consumer) bus, for which the active and reactive powers are known

- A PV type (generator) bus for which the active power and voltage magnitude are known. For this type of bus, limit values for the reactive power are defined: $Q_{min} \leq Q \leq Q_{max}$
- The slack bus, for which the magnitude and angle of the voltage are known, and the angle is often considered as 0, reference value for the entire system.

It can be seen that, with these assumptions, solving directly the load flow by using eq. (8.1) is no longer possible, because the value of the current injection in the slack bus is not known.

Branch data: *branch type, (line, transformer, series reactances, shunt capacitors), connection buses and electrical parameters (resistance, reactance, conductance, susceptance),* computed used manufacturer catalogue specifications and other system specifics (transformer tap and type of adjustment, line length, number of parallel branches or transformers) and equivalent model (with lumped parameters, with distributed parameters, choice of use of transversal parameters).

Computation of the bus admittance matrix

The bus admittance matrix $[\underline{Y}_n]$ is a square matrix, with a size equal to the number of independent buses in the system. If the analyzed system does not contain ideal transformers, the $[\underline{Y}_n]$ matrix is symmetrical, and its elements are computed as follows: [Eremia 85]:

- A diagonal element, $\underline{Y}_{ii}$, is the sum of the admittances of the branches connected to the *i* bus.
- A non-diagonal element, $\underline{Y}_{ij}$, is the value of the admittance of the branch connected between buses *i* and *j*, taken with the minus sign. If there is no branch defined between the *i* and *j buses,* the value from the matrix will be considered 0.

If the system contains ideal transformers,

- A diagonal element, $\underline{Y}_{ii}$, is the sum of the admittances of the branches electrically connected to the *i* bus, and of the branches magnetically connected to the *i* bus, multiplied by the transformer's turns ratio:

$$\underline{Y}_{ii} = \sum_{k \in L_i} N_{ki}^2 \cdot \underline{y}_k \tag{8.2}$$

where L_i is the multitude of branches connected to bus *i*, and N_{ki} is the turn ratio f the transformer from the *k* branch connected to bus *i*. If branch *k* is electrically connected to bus i, then $N_{ki}=1$.

- A diagonal element, $\underline{Y}_{ii}$, is the sum of the admittances of the branches connected to the *i* bus, multiplied by the transformer's turns ratio. If the magnetic connection is in bus *i,* then the complex conjugate of the transformer ratio is used:

$$\underline{Y}_{ij} = -N_{ki}^* \cdot \underline{y}_k \qquad \underline{Y}_{ji} = -N_{ki} \cdot \underline{y}_k \tag{8.3}$$

If the transformers have complex turn ratios, then the bus admittance matrix is no longer symmetrical.

Solving the load flow

The state variables are computed: bus voltage magnitudes and angles for the PQ buses, reactive power injections and bus voltage angles for the PV buses.

Computation of the auxiliary variables of the load flow

- Active and reactive branch power flows
- Branch power losses
- The active and reactive power injection at the slack bus.
- Branch voltage drops
- Branch current flows

8.2. The Newton-Raphson load flow algorithm

In its basic, analytical form, applied to any type of problem, the Newton-Raphson method is the equivalent of the Newton method for solving one-variable non-linear equations written as:

$$f(x) = 0 \tag{8.4}$$

If the Taylor expansion near the current approximation of the solution x^t is used, where only the terms up to the first order are kept, and the real solution is $x^* = x^t + \Delta x^t$, then:

$$f(x^*) = f(x^t + \Delta x^t) \approx f(x^t) + f^{'}(x^t) \cdot \Delta x^t \tag{8.5}$$

x^* being the solution of (8.4), (8.5) gives the correction Δx^t which, applied to the current approximation x^t, will lead to a new approximation, x^{t+1}, which deviates from the solution x^* because the non-linear element from the Taylor expansion were ignored. This means a linearization of the problem.

$$x^{t+1} = x^t + \Delta x^t = x^t - \frac{f(x^t)}{f^{'}(x^t)} \tag{8.6}$$

In a graphic representation (Fig. 8.1), the $f^{'}(x)$ derivative is the slope of the tangent to the $y = f(x)$ curve, in the current point.

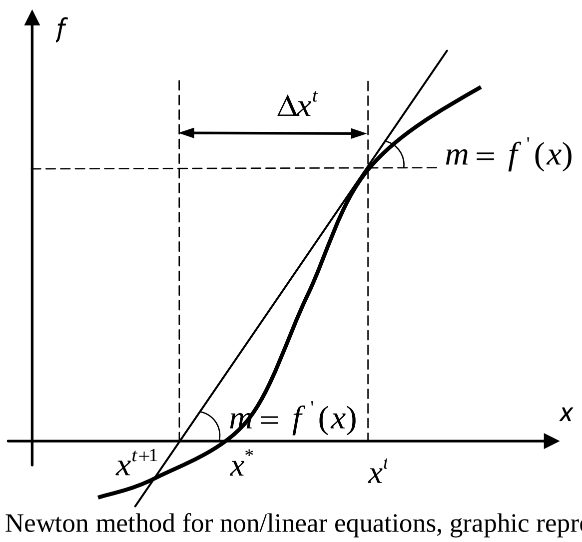

Fig. 8.1 – The Newton method for non/linear equations, graphic representation

For a *n*-sized equation system with *n* unknown variables, eq. (8.4) can be written as:

$$\Im([x]) = 0 \tag{8.7}$$

where

$[x]=[x_1 \; x_2 \; ... \; x_n]^T$ vector of unknown variables,

$\Im=[f_1 \; f_2 \; ... \; f_n]^T$ vector of the functions which define the equations system, computed in point $[x]$.

(5) and (6) are then written as:

$$\Im([x^*]) = \Im([x]^t + [\Delta x]^t) \approx \Im([x]^t) + J([x]^t) \cdot [\Delta x]^t \tag{8.8}$$

and

$$[x]^{t+1} = [x]^t + [\Delta x]^t = [x]^t - J^{-1}([x]^t) \cdot \Im([x]^t) \tag{8.9}$$

$J([x]) \overset{not}{=} J$ is a n-sized square matrix, the Jacobian matrix, where any $J_{ik} = \partial f_i / \partial x_k$ is the derivative f_i at point x_k:

$$[J] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{8.10}$$

Using the Newton-Raphson method in load flow calculations requires writing the load flow equations in an (8.7) type equivalent form.

Given a system with *N* independent buses, where bus *e* was designed as slack bus, the complex power associated to an *i* bus can be written as:

$$\underline{S}_i = P_i + j \cdot Q_i = \underline{U}_i \cdot \underline{J}_i^* = \underline{U}_i \cdot \sum_{k=1}^{N} \underline{Y}_{ik}^* \cdot \underline{U}_k^*, \quad i = 1..N, i \neq e \tag{8.11}$$

If the elements form the bus admittance matrix are written as $\underline{Y}_{ik} = G_{ik} + j \cdot B_{ik}$, and the bus voltages are written as $\underline{U}_i = U_i \cdot e^{j\theta_i}$ (algebraic and polar form), the following expressions for the active an reactive power can be determined:

$$\begin{aligned} P_i &= G_{ii} \cdot U_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^{N} U_i \cdot U_k \cdot [G_{ik} \cdot \cos(\theta_i - \theta_k) + B_{ik} \cdot \sin(\theta_i - \theta_k)] \\ Q_i &= -B_{ii} \cdot U_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^{N} U_i \cdot U_k \cdot [B_{ik} \cdot \cos(\theta_i - \theta_k) - G_{ik} \cdot \sin(\theta_i - \theta_k)] \end{aligned} \tag{8.12}$$

$$i = 1..N, i \neq e$$

For given approximate values of the voltages $\underline{U}_i$ if all the buses in the systems are of PQ type, the P_i and Q_i values computed with (8.12) will deviate from the real values, given as input data, P_i^{imp} and Q_i^{imp} with the quantities ΔP_i, and ΔQ_i. If the accurate solution were to be found, ΔP_i, and ΔQ_i would be null. But, for approximations of the accurate solutions, they are not null. To compensate for the deviations, the voltage magnitudes and angles ΔU_i and $\Delta\theta_i$, should be corrected,. The ΔP_i and ΔQ_i deviations can be written as:

$$\begin{aligned}\Delta P_i &= \sum_{k=1}^{N}\left[\frac{\partial P_i}{\partial\theta_k}\cdot\Delta\theta_k + \frac{\partial P_i}{\partial U_k}\cdot U_k\cdot\frac{\Delta U_k}{U_k}\right] \\ \Delta Q_i &= \sum_{k=1}^{N}\left[\frac{\partial Q_i}{\partial\theta_k}\cdot\Delta\theta_k + \frac{\partial Q_i}{\partial U_k}\cdot\cdot U_k\cdot\frac{\Delta U_k}{U_k}\right]\end{aligned}\qquad i=1..N, i\neq e \tag{8.13}$$

For easing the following computations, the voltage magnitude corrections ΔU_k are replaced with $\Delta U_k/U_k$, and expression (8.13) is rewritten as:

$$\begin{aligned}\Delta P_i &= H_{ii}\cdot\Delta\theta_i + N_{ii}\cdot\frac{\Delta U_i}{U_i} + \sum_{\substack{k=1\\k\neq i}}^{N}\left(H_{ik}\cdot\Delta\theta_k + N_{ik}\cdot\frac{\Delta U_k}{U_k}\right) \\ \Delta Q_i &= J_{ii}\cdot\Delta\theta_i + L_{ii}\cdot\frac{\Delta U_i}{U_i} + \sum_{\substack{k=1\\k\neq i}}^{N}\left(J_{ik}\cdot\Delta\theta_k + L_{ik}\cdot\frac{\Delta U_k}{U_k}\right)\end{aligned}\qquad i=1..N,\ i\neq e \tag{8.14}$$

In matrix form, this is written as:

$$\left[\begin{array}{cc|c|cc|c|cc} H_{11} & N_{11} & \multirow{2}{*}{$\cdots$} & H_{1k} & N_{1k} & \multirow{2}{*}{$\cdots$} & H_{1N} & N_{1N} \\ J_{11} & L_{11} & & J_{1k} & L_{1k} & & J_{1N} & L_{1N} \\ \hline \multicolumn{2}{c|}{\cdots} & & \multicolumn{2}{c|}{\cdots} & & \multicolumn{2}{c}{\cdots} \\ \hline H_{i1} & N_{i1} & \cdots & H_{ik} & N_{ik} & \cdots & H_{iN} & N_{iN} \\ J_{i1} & L_{i1} & & J_{ik} & L_{ik} & & J_{iN} & L_{iN} \\ \hline \multicolumn{2}{c|}{\cdots} & & \multicolumn{2}{c|}{\cdots} & & \multicolumn{2}{c}{\cdots} \\ \hline H_{N1} & N_{N1} & \cdots & H_{Nk} & N_{Nk} & \cdots & H_{NN} & N_{NN} \\ J_{N1} & L_{N1} & & J_{Nk} & L_{Nk} & & J_{NN} & L_{NN} \end{array}\right] * \left[\begin{array}{c} \Delta\theta_1 \\ \Delta U_1/U_1 \\ \hline \cdots \\ \hline \Delta\theta_i \\ \Delta U_i/U_i \\ \hline \cdots \\ \hline \Delta\theta_N \\ \Delta U_N/U_N \end{array}\right] = \left[\begin{array}{c} \Delta P_1 \\ \Delta Q_1 \\ \hline \cdots \\ \hline \Delta P_i \\ \Delta Q_i \\ \hline \cdots \\ \hline \Delta P_N \\ \Delta Q_N \end{array}\right] \tag{8.15}$$

Expressions (8.14) and (8.15) describe an equations systems with *2*(N-1)* unknowns, the voltage magnitude ($\Delta U_k/U_k$) and angle ($\Delta\theta_k$) correction. The terms denoted with ***H, J, L*** in ***N*** in (8.15) form the Jacobian matrix. If the system contains PV buses, the size of the Jacobian, and consequently of the equations system (8.15), will be reduced. For the PV buses, the lines corresponding to the J_{ik} an L_{ik} terms will not appear, the right side of the equations system will not contain the reactive power deviations ΔQ_i, and only the voltage angle deviations $\Delta\theta_i$ will be computed.

The elements of the Jacobian matrix are computed as:

$$H_{ii} = \frac{\partial P_i}{\partial \theta_i} = -\sum_{\substack{k=1\\k\neq i}}^{N} U_i \cdot U_k \cdot \left[G_{ik} \cdot \sin(\theta_i - \theta_k) - B_{ik} \cdot \cos(\theta_i - \theta_k)\right] = -B_{ii} \cdot U_i^2 - Q_i$$

$$H_{ik} = \frac{\partial P_i}{\partial \theta_k} = U_i \cdot U_k \cdot \left[G_{ik} \cdot \sin(\theta_i - \theta_k) - B_{ik} \cdot \cos(\theta_i - \theta_k)\right]$$

$$N_{ii} = \frac{\partial P_i}{\partial U_i} \cdot U_i = \left[2 \cdot G_{ii} \cdot U_i - \sum_{\substack{k=1\\k\neq i}}^{N} U_k \cdot \left[G_{ik} \cdot \cos(\theta_i - \theta_k) + B_{ik} \cdot \sin(\theta_i - \theta_k)\right]\right] \cdot U_i =$$
$$= P_i + G_{ii} \cdot U_i^2$$

$$N_{ik} = \frac{\partial P_i}{\partial U_k} \cdot U_k = U_i \cdot U_k \cdot \left[G_{ik} \cdot \cos(\theta_i - \theta_k) + B_{ik} \cdot \sin(\theta_i - \theta_k)\right]$$

$$J_{ii} = \frac{\partial Q_i}{\partial \theta_i} = \sum_{\substack{k=1\\k\neq i}}^{N} U_i \cdot U_k \cdot \left[B_{ik} \cdot \sin(\theta_i - \theta_k) + G_{ik} \cdot \cos(\theta_i - \theta_k)\right] = P_i - G_{ii} \cdot U_i^2$$

$$J_{ik} = \frac{\partial Q_i}{\partial \theta_k} = -U_i \cdot U_k \cdot \left[B_{ik} \cdot \sin(\theta_i - \theta_k) + G_{ik} \cdot \cos(\theta_i - \theta_k)\right] = -N_{ik}$$

$$L_{ii} = \frac{\partial Q_i}{\partial U_i} \cdot U_i = -\left[2 \cdot B_{ii} \cdot U_i + \sum_{\substack{k=1\\k\neq i}}^{N} U_k \cdot \left[B_{ik} \cdot \cos(\theta_i - \theta_k) - G_{ik} \cdot \sin(\theta_i - \theta_k)\right]\right] \cdot U_i =$$
$$= Q_i - B_{ii} \cdot U_i^2$$

$$L_{ik} = \frac{\partial Q_i}{\partial U_k} = -U_i \cdot U_k \cdot \left[B_{ik} \cdot \cos(\theta_i - \theta_k - G_{ik} \cdot \sin(\theta_i - \theta_k)\right] = H_{ik}$$

By solving the expression (8.14), the corrections for the voltage magnitudes and angles are computed and the current approximations of the state variables are updated:

$$\begin{aligned} U_i^{t+1} &= U_i^t + \Delta U_i^t \qquad & i \equiv PQ \\ \theta_i^{t+1} &= \theta_i^t + \Delta\theta_i^t \qquad & i \equiv PQ \cup PU \end{aligned} \tag{8.16}$$

The iterative process continues by recomputing the Jacobian matrix and new voltage magnitude and angle corrections until a stopping criterion is met. Such a criterion can be meeting the condition that, for two consecutive iterations, the deviation of the computed active and reactive bus power injections falls under a given value

The Newton-Raphson method is widely used for solving load flow problems because of its advantages:

- small number of iterations
- high precision

However, the method has some important drawbacks:

- high computational effort per iteration, due to the recomputing of the Jacobian ant its inverse
- convergence problems, when the initial approximation of the solution is too far from the solution or the problem is ill-conditioned

To overcome these disadvantages, simplified variants of the Newton-Raphson method are used:

- the decoupled Newton-Raphson method
- the fast decoupled Newton-Raphson method

However, while they offer a decrease in number of calculations per iteration, this advantage is impaired by the higher number of iterations to reach convergence and by the errors in final solution arising from the simplifying assumptions taken.

The pseudocode for the Newton-Raphson algorithm is given in Box 1 [Gavrilaş 08].

Box 1 – The Newton-Raphson algorithm

1. Provide the input data;
2. Provide the initial approximation for the state variables $\underline{U}_{i(0)}$, *i=1..N, i ≠ e* and initialize the iterative process (*it=0*);
3. For all PQ and PV buses: *i=1..N, i ≠ e:*
    - 3.1. Compute the bus active and reactive powers, P_i and Q_i, with (5)
    - 3.2. Compute the active powers deviations $\Delta P_i = P_i^{imp} - P_i$
    - 3.3. For each PV bus *i*:
        - 3.3.1.1. If $Q_i \prec Q_i^{\min}$, temporarily make bus *i* a PQ bus, with $Q_i^{imp} = Q_i^{\min}$ and go to step 3.3.4;
        - 3.3.1.2. If $Q_i \succ Q_i^{\max}$, temporarily make bus *i* a PQ bus, with $Q_i^{imp} = Q_i^{\max}$ and go to step 3.3.4;
        - 3.3.1.3. If $Q_i^{\min} \prec Q_i \prec Q_i^{\max}$, go to step 3.5;
        - 3.3.1.4. Add a equation for the reactive power, for the new PQ bus and go to step 3.4;
    - 3.4. Compute the active powers deviation $\Delta Q_i = Q_i^{imp} - Q_i$;
    - 3.5. Go to next bus *i* , go to step 3.1, until all buses are visited.
4. The stopping criterion. If $\max\limits_{i \in PQ,PU} (\Delta P_i) \prec \varepsilon_p$ and $\max\limits_{i \in PQ} (\Delta Q_i) \prec \varepsilon_Q$, go to step 8;
5. Compute the jacobian matrix, solve the (8) equations system, and find the corrections $\Delta U_k / U_k$ and $\Delta \theta_k$;
6. Compute the new approximation for the bus voltage magnitude and angle;
7. Iterations count check:
    - If $it = IT_{max}$, stop the algorithm with warning error
    - If $it < IT_{max}$, proceed with a new iteration and go to step 3.
8. With the found solution, compute branch power flows and other auxiliary load flow results.

8.3. The decoupled Newton-Raphson method

Based on the assumption that for most power systems, there is a low correlation between the pairs of components ***P-U*** and ***Q-θ,*** which is equivalent to small values for the $\partial P/\partial U$ and $\partial Q/\partial \theta$ derivatives, which can be eliminated from the mathematical model. Thus, the *N* and *J* components of the Jacobian matrix are null, and the (13) expression is rewritten as:

$$\Delta P_i = \sum_{\substack{k=1 \\ k\neq i}}^{N} H_{ik} \cdot \Delta\theta_k \qquad\qquad i = 1..N,$$
$$\Delta Q_i = \sum_{\substack{k=1 \\ k\neq i}}^{N} L_{ik} \cdot \frac{\Delta U_k}{U_k} \qquad\qquad i \neq e \qquad (8.17)$$

This means the effective separation or decoupling of ***P-θ*** and ***Q-U***, resulting in two distinct equations systems, each with ***N-1*** equations and ***N-1*** unknown variables (corrections for voltage magnitude and voltage angle respectively). The H_{ik} and L_{ik} coefficients from (8.17) are computed using the complete expressions from the Newton-Raphson method. For a faster convergence, the ***P-θ*** equation system is solved first, finding the new voltage angle corrections $\Delta\theta_i$. With these, new L_{ik} coefficients are computed and then the ***Q-U*** system from (8.17) is solved, finding the new ΔU_i voltage magnitude corrections. This allows speeding the time of calculation for an iteration, but this advantage is compensated by the higher number of iterations required for reaching convergence, compared with the full Newton-Raphson method.

2.4. The fast decoupled Newton-Raphson method

The (8.17) expressions can be further simplified if the following assumptions are considered:

- $\theta_i - \theta_k \approx 0$, because the angle difference between two neighboring buses is negligible. Thus, in (17):

$$\sin(\theta_i - \theta_k) \approx 0 \qquad \cos(\theta_i - \theta_k) \approx 1 \qquad (8.18)$$

- $G_{ik} << B_{ik}$ – because the branch conductances are much smaller than the susceptance, thus

$$G_{ik} = 0 \qquad (8.19)$$

- in the H_{ii} and L_{ii} coefficients from the Jacobian matrix, it is considered that Q_i << $B_{ii} \cdot U_i^2$, thus

$$Q_i = 0 \qquad (8.20)$$

With these assumptions, considering also the decoupling of ***P-θ*** and ***Q-U*** equations, the H, N, L and J coefficients from the Jacobian matrix are computed as:

$$\begin{aligned} H_{ii} &= L_{ii} \approx -B_{ii} \cdot U_i^2 & N_{ii} &= J_{ii} \approx 0 \\ H_{ik} &= L_{ik} \approx -B_{ik} \cdot U_i \cdot U_k & N_{ik} &= J_{ik} \approx 0 \end{aligned} \qquad (8.21)$$

The B_{ik} coefficients are further simplified as follows:

- in the P-θ equations, from B_{ik} are eliminated all branches which may have a influence on the voltage levels (transformer taps) and on the reactive power flow (reactance coils or capacitor banks) and the bus voltage is considered with its rated value $U_{k,n}$. From the remaining branches, the real components of the admittance (*R* and *G*) are also discarded. Thus, for the P-θ equations:

$$B_{ik}^{'} \overset{not}{=} -B_{ik} \cdot U_{k,n} \tag{8.22}$$

- in the Q-U, equations, all the branches which could affect the active power flows. Thus:

$$B_{ik}^{''} \overset{not}{=} -B_{ik} \tag{8.23}$$

With these simplifications, (8.17) is rewritten as:

$$\begin{aligned} \Delta P_i &= \sum_{\substack{k=1 \\ k\neq i}}^{N} B_{ik}^{'} \cdot \Delta\theta_k \\ \Delta Q_i &= \sum_{\substack{k=1 \\ k\neq i}}^{N} B_{ik}^{''} \cdot \Delta U_k \end{aligned} \tag{8.24}$$

In (8.24), the $B_{ik}^{'}$ and $B_{ik}^{''}$ coefficients are constant, which means that the Jacobian matrix will remain unchanged during the iterative process, if PV buses are not temporarily changed to PQ buses. In this case, the number of ΔQ_i equations changes and, only then, the Jacobian matrix must be recomputed.

2.5. Per-unit values

In electrical networks with two or more voltage levels, values as branch impedances or voltage values cannot be compared on a common reference. In this case, the per-unit system is often used. Converting absolute values into per-unit values is, basically, a normalization process, using a base value of choice.

$$\text{per unit value} = \frac{\text{absolute value}}{\text{base value}} \tag{8.25}$$

The value of the numerators in expression (8.25) may be a complex number; however, the denominator (the base value) is a positive real number.

The per-unit value is dimensionless. In the literature, as a convention a parameter for which its value is given as per-unit (p.u.), it will be affected by the symbol (*).

In power systems there are four base quantities required to define a per-unit system. These are: power ($\underline{S}$), voltage magnitude ($\underline{U}$), current ($\underline{I}$) and impedance ($\underline{Z}$). For these, the known basic interdependences are defined:

$$\underline{U} = \underline{Z} \cdot \underline{I} \quad ; \quad \underline{S} = \underline{U} \cdot \underline{I}^* \tag{8.26}$$

Given two of them, all others can be defined using these two. Usually, the base voltage U_B and power S_B are chosen. The voltage angle is dimensionless. Base current and impedance follow as:

$$I_B = \frac{S_B}{U_B}, \quad Z_B = \frac{U_B^2}{S_B} \quad \text{or} \quad Y_B = \frac{S_B}{U_B^2} = \frac{1}{Z_B} \tag{8.27}$$

As a general rule, the base power has a unique value at system level, while the base voltage is recomputed using the transformers' ratio.

The per-unit system is useful particularly when representing transformers. If as base value for voltages, the rated voltages of the transformer are chosen, then the ratio is no longer needed, because the voltages in p.u. are equal.

Advantages of the per-unit system

- Ohm's Law is the same in the per-unit system: if $\underline{U} = \underline{Z} \cdot \underline{I}$ and $U_B = Z_B \cdot I_B$, then

$$\underline{U}_{p.u.} = \underline{Z}_{p.u.} \cdot \underline{I}_{p.u.} \tag{8.28}$$

- The three-phase and the single-phase base values of voltages and powers are related as:

$$S_B^{3-ph} = 3 \cdot S_B^{1-ph} \qquad U_B^{LL} = \sqrt{3} \cdot U_B^{LG} \tag{8.29}$$

- The impedance and current have the same values when computed using single-phase or three-phase values:

$$I_B = \frac{S_B^{3f}}{\sqrt{3} \cdot U_B^{f\text{-}f}} = \frac{3 \cdot S_B}{\sqrt{3} \cdot (\sqrt{3} \cdot U_B)} = \frac{S_B}{U_B} = I_B$$

$$Z_B = \frac{(U_B^{f\text{-}f})^2}{S_B^{3f}} = \frac{(\sqrt{3} \cdot U_B)^2}{3 \cdot S_B} = \frac{U_B^2}{S_B} = Z_B \tag{8.30}$$

- The per-unit one phase and three-phase values are equal:

$$\underline{S}_{p.u.}^{3ph} = \frac{\underline{S}^{3ph}}{S_B^{3ph}} = \frac{3 \cdot \underline{S}}{3 \cdot S_B} = \underline{S}_{p.u.} \qquad \underline{U}_{p.u.}^{LL} = \frac{\underline{U}^{LL}}{U_B^{LL}} = \frac{\sqrt{3} \cdot \underline{U}}{\sqrt{3} \cdot U_B} = \underline{U}_{p.u.} \tag{8.31}$$

Base change

There are cases when it is necessary to change the base value, for instance when a system contains transformers and, thus, more than one voltage areas, or when the transformers' basic parameters are given in producers' catalogues using different base values (absolute values, percent etc). In these situations, it is necessary to recompute the per-unit values of impedances or admittances.

For the same absolute value of the impedance:

$$\underline{Z} = \underline{Z}_{p.u.}^{old} \cdot Z_B^{old} = \underline{Z}_{p.u.}^{new} \cdot Z_B^{new} \tag{8.32}$$

which gives:

$$\underline{Z}_{p.u.}^{new} = \underline{Z}_{p.u.}^{old} \cdot \frac{Z_B^{old}}{Z_B^{new}} \Leftrightarrow \underline{Z}_{p.u.}^{new} = \underline{Z}_{p.u.}^{old} \cdot \frac{(U_B^{old})^2}{(U_B^{new})^2} \cdot \frac{S_B^{new}}{S_B^{old}} \tag{8.33}$$

The same applies for admittances.

Modeling transformers in the per-unit system

For the two-winding transformer, when isolated from the system, its per-unit parameters are computed using as base values the rated power $S_B = S_n$ and one of the rated voltages $U_{B1} = U_n^{HV}$ or $U_{B2} = U_n^{LV}$

Thus, the base impedances and admittances can be computed as:

$$Z_{B1} = \frac{U_{B1}^2}{S_{B1}} \qquad \text{or} \qquad Y_{B1} = \frac{S_{B1}}{U_{B1}^2}$$

$$\tag{8.34}$$

$$Z_{B2} = \frac{U_{B2}^2}{S_{B2}} \qquad \text{or} \qquad Y_{B2} = \frac{S_{B2}}{U_{B2}^2}$$

The per-unit parameters of the transformer will be computed as:

$$R_{p.u.} = \frac{R^{HV}}{Z_{B1}} = \frac{\Delta P_{sc} \cdot (U_n^{HV})^2}{S_n^2} \cdot \frac{S_n}{(U_n^{HV})^2} = \frac{\Delta P_{sc} \cdot (U_n^{LV})^2}{S_n^2} \cdot \frac{S_n}{(U_n^{LV})^2} = \frac{R^{LV}}{Z_{B2}} = \frac{\Delta P_{sc}}{S_n}$$

$$X_{p.u.} = \frac{X^{HV}}{Z_{B1}} = \frac{u_{sc} \cdot (U_n^{HV})^2}{100 \cdot S_n} \cdot \frac{S_n}{(U_n^{HV})^2} = \frac{u_{sc} \cdot (U_n^{LV})^2}{100 \cdot S_n} \cdot \frac{S_n}{(U_n^{LV})^2} = \frac{X^{LV}}{Z_{B2}} = \frac{u_{sc}}{100} \tag{8.35}$$

$$G = \frac{G^{IT}}{Y_{B1}} = \frac{\Delta P_{Fe}}{(U_n^{IT})^2} \cdot \frac{(U_n^{IT})^2}{S_n} = \frac{\Delta P_{Fe}}{(U_n^{JT})^2} \cdot \frac{(U_n^{JT})^2}{S_n} = \frac{G^{JT}}{Y_{B2}} = \frac{\Delta P_{Fe}}{S_n}$$

$$B_{p.u.} = \frac{B^{HV}}{Y_{B1}} = \frac{i_0 \cdot S_n}{100 \cdot (U_n^{HV})^2} \cdot \frac{(U_n^{HV})^2}{S_n} = \frac{i_0 \cdot S_n}{100 \cdot (U_n^{LV})^2} \cdot \frac{(U_n^{LV})^2}{S_n} = \frac{B^{LV}}{Y_{B2}} = \frac{i_0}{100}$$

If the rated voltages of the transformer are chosen as base voltages:

$$r \cdot \frac{U_{B2}}{U_{B1}} = \frac{U_n^{LV}}{U_n^{HV}} = N \tag{8.36}$$

then the per-unit values of the parameters given by (8.35) are the same, regardless of the base voltage used, and, consequently, in the per-unit diagram, the ratio of the transformer is eliminated.

In eq. (8.36), if N is expressed using the number of turns, the coefficient r has one of the values from Table 8.1.

Table 8.1- The influence of the vector group of the transformer on the value of the transformer ratio used for converting to the per-unit system

r	HV winding vector group	LV winding vector group
1	Y	Y
1	Δ	Δ
$1/\sqrt{3}$	Δ	Y
$\sqrt{3}$	Y	Δ

A basic convention for applying the conversion in per-unit values of a given system

1. Build the one-line diagram of the system
2. Choose a base value for the power (a multiple of 10 MVA or 10 kVA is recommended).
3. Place the rated ratios of transformers on the one-line diagram.
4. Identify the voltage areas of the system, based on the transformer winding voltages.
5. Choose the base voltage in the first area (the rated voltage in the area is the recommended value).
6. Compute the base voltages in the other areas, using the already known base voltages and transformer ratios.
7. Compute the base impedance in each area.
8. Compute the per-unit values of impedances, powers and voltages in each area.
9. Build the one-line diagram in p.u. and eliminate the transformers.
10. Run the analysis and reconvert the results in absolute values, if necessary.

8.6. Example: Using a load flow Newton-Raphson algorithm for power system management

In the following example, the Newton-Raphson load flow algorithm will be used on the IEEE 14-bus system [web pstca] for testing if an important line can be disconnected for maintenance reasons. The one-line diagram of the system is presented in Fig. 8.2.

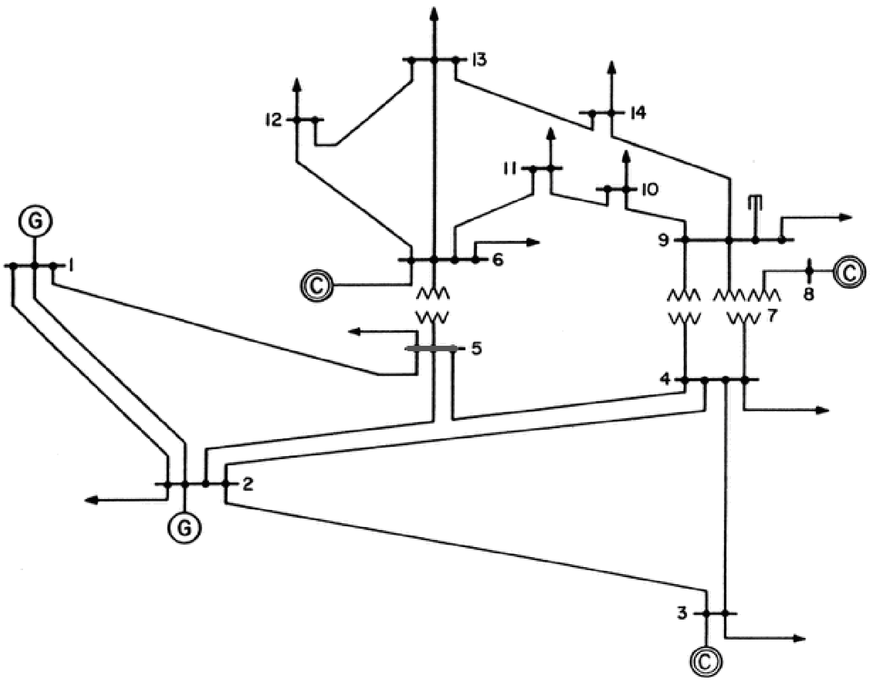

Fig. 8.2 – The one line diagram of the IEEE 14-bus system

As input data, the load flow algorithm requires the branch parameters given in Table 8.2 and Table 8.3.

Table 8.2 - Bus parameters for the IEEE 14-bus system

bus no.	bys type	initial bus voltage magnitude [p.u.]	initial bus voltage angle [radians]	consumed active power [MW]	consumed reactive power [MVAr]	generated active power [MW]	generated reactive power [MVAr]	max. reactive power limit [MVAr]	min. reactive power limit [MVAr]
1	slack	1.06	0	0	0	0	0	0	0
2	PV	1.045	0	-21.7	-12.7	40	0	50	-40
3	PV	1.01	0	-94.2	-19	0	0	40	0
4	PQ	1	0	-47.8	3.9	0	0	0	0
5	PQ	1	0	-7.6	-1.6	0	0	0	0
6	PV	1.07	0	-11.2	-7	0	0	24	-6
7	PQ	1	0	0	0	0	0	0	0
8	PV	1.09	0	0	0	0	0	24	-6
9	PQ	1	0	-29.5	-16.6	0	0	0	0
10	PQ	1	0	-9	-5.8	0	0	0	0
11	PQ	1	0	-3.5	-1.8	0	0	0	0
12	PQ	1	0	-6.1	-1.6	0	0	0	0
13	PQ	1	0	-13.5	-5.8	0	0	0	0
14	PQ	1	0	-14.9	-5	0	0	0	0

Table 8.3 - Branch parameters for the IEEE 14-bus system

bus i	bus j	branch type	branch resistance [p.u.]	branch reactance [p.u.]	branch susceptance [p.u.]	transformer tap
1	2	0	0.01938	0.05917	0.0528	1
1	5	0	0.05403	0.22304	0.0492	1
2	3	0	0.04699	0.19797	0.0438	1
2	4	0	0.05811	0.17632	0.0374	1
2	5	0	0.05695	0.17388	0.034	1
3	4	0	0.06701	0.17103	0.0346	1
4	5	0	0.01335	0.04211	0.0128	1
4	7	1	0	0.20912	0	0.978
4	9	1	0	0.55618	0	0.969
5	6	1	0	0.25202	0	0.932
6	11	0	0.09498	0.1989	0	1
6	12	0	0.12291	0.25581	0	1
6	13	0	0.06615	0.13027	0	1
7	8	0	0	0.17615	0	1
7	9	0	0	0.11001	0	1
9	10	0	0.03181	0.0845	0	1
9	14	0	0.12711	0.27038	0	1
10	11	0	0.08205	0.19207	0	1
12	13	0	0.22092	0.19988	0	1
13	14	0	0.17093	0.34802	0	1

In Table 8.2, the bus powers are given in absolute quantities, which will be transformed subsequently in per-unit values, using a base power of 100 MVA. The initial voltages columns contain the initial approximation of the bus voltages and angles. The magnitudes are given in per-unit values, while the angles are given in radians.

For the slack bus and PV buses, the initial approximation is the value of the voltage imposed at the bus. Also for the PV buses, the limit values of the reactive powers are specified. If any of these is violated, the bus will be made a PQ bus and its voltage will change.

In Table 8.3, all the parameters are given in per-unit values. The branch conductances are considered 0. A branch type of 1 denotes a line, for which the ratio is always 1, while a type 0 branch is a transformer, for which the current tap is given in percent.

In the reference scenario, the Newton-Raphson method converges in 5 iterations, for a precision of 0.001. The results are the bus voltages, as magnitude and angle, the power injection at the slack bus, and the reactive powers at the PV buses, presented in Table 8.4. Based on these quantities, branch power flows, currents and voltage drops and power losses can be computed.

For the used load pattern, the system needs 235 MW of active power, which will be imported through the slack bus, and will have a surplus of 19 MVAr of reactive power, which will flow out through the slack bus. Bus 6 is transformed into a PQ bus, its maximum reactive generation limit being exceeded in order to keep the voltage level near to 1.07 p.u. The voltages in the PQ buses decrease progressively as the buses are more far away from the slack bus. All the voltage magnitudes are in the range of +/- 5% tolerance from the rated voltage, which in per-unit values is 1. The voltage angles are progressively lagging.

Table 8.4 – Load flow results - the IEEE 14-bus system

bus no.	bus type	bus voltage magnitude [p.u.]	bus voltage angle [radians]	bus active power [MW]	bus reactive power [MVAr]	max. reactive power limit	min. reactive power limit
1	slack	1.06	0	234.8447	-18.7697	0	0
2	PV	1.045	-0.101	18.3	36.74956	50	-40
3	PV	1.01	-0.259	-94.2	12.28612	40	0
4	PQ	1.011	-0.208	-47.8	3.9	0	0
5	PQ	1.014	-0.178	-7.6	-1.6	0	0
6	PQ	1.069	-0.293	-11.2	24.0005	24	-6
7	PQ	1.046	-0.269	-1.1E-06	1.65E-07	0	0
8	PV	1.09	-0.269	1.67E-14	23.49243	24	-6
9	PQ	1.027	-0.301	-29.5	-16.6	0	0
10	PQ	1.026	-0.305	-9	-5.8	0	0
11	PQ	1.043	-0.301	-3.5	-1.8	0	0
12	PQ	1.050	-0.310	-6.1	-1.6	0	0
13	PQ	1.043	-0.311	-13.5	-5.8	0	0
14	PQ	1.013	-0.326	-14.9	-5	0	0

The next step is to disconnect the line scheduled for maintenance. An important line in the system is chosen, namely the line linking buses 1 and 2, which is one of the two lines connected to the slack bus, through which the entire system is supplied.

Running again the algorithm, with the bus loads from Table 8.2, the load flow does not converge. After 100 iterations, the results of the algorithm are the ones presented in Table 8.5. This means that, for the given load profile, it is not possible to keep the system running if the chosen line is disconnected.

Table 8.5 – Load flow results - the IEEE 14-bus system with line 1-2 disconnected

bus no.	bus type	bus voltage magnitude [p.u.]	bus voltage angle [radians]	bus active power [MW]	bus reactive power [MVAr]	max. reactive power limit	min. reactive power limit
1	slack	1.06	0	-19.787	2.843	0	0
2	PQ	-0.066	0	0.425	1.657	50	-40
3	PV	0.115	0	0.003	0.134	40	0
4	PQ	0.029	0	2.954	-0.628	0	0
5	PQ	5.185	0	276.983	886.080	0	0
6	PQ	3.837	0	46.793	285.354	24	-6
7	PQ	-2.186	0.97	0.000	0.000	0	0
8	PQ	12.46	1.105	0.000	767.506	24	-6
9	PQ	-0.066	0	-0.019	0.108	0	0
10	PQ	-0.025	0	0.019	0.005	0	0
11	PQ	-0.002	0	-0.027	-0.026	0	0
12	PQ	0.005	0	-0.061	-0.016	0	0
13	PQ	-0.000	0	0.008	0.005	0	0
14	PQ	0.120	0	0.051	0.054	0	0

In this case, the dispatcher must make a choice. Generally, some available choices would be:

- To supply bus 2 from a different source
- To reduce the load in the system by disconnecting important consumers.
- To reschedule the maintenance to a time when the load profile in the system allows it to be supplied only through line 1-3.

Solution 1: Changing the configuration of the system so that bus 2 is supplied from an alternative source.

Analyzing the one-line diagram, it can be seen that no alternative supply source exists.

Solution 2: Disconnect loads from the system.

Analyzing the load profile from Table 8.2, bus 3 has the highest load, which is over a third of the system's entire imported power.

The load and generation from the bus are zeroed, the bus is transformed into a PQ-type bus and the load flow is executed again. The results are given in Table 8.6. This time, bus 3 is changed into a PQ bus, but the algorithm converges in 3 iterations, which means that disconnecting the load in bus 3 makes the maintenance of the 1-2 line possible.

Table 8.6 – Load flow results - the IEEE 14-bus system with line 1-2 disconnected and the load from bus 3 zeroed

bus no.	bus type	bus voltage magnitude [p.u.]	bus voltage angle [radians]	bus active power [MW]	bus reactive power [MVAr]	max. reactive power limit	min. reactive power limit
1	slack	1.060	0.000	137.384	5.748	0	0
2	PV	1.045	0.000	18.300	25.854	50	-40
3	PQ	1.034	0.000	0.000	0.000	40	0
4	PQ	1.015	0.000	-47.800	3.900	0	0
5	PQ	1.013	0.000	-7.600	-1.600	0	0
6	PV	1.070	0.000	-11.200	17.034	24	-6
7	PQ	1.048	0.000	0.000	0.000	0	0
8	PQ	1.090	0.000	0.000	22.660	24	-6
9	PQ	1.029	0.000	-29.500	-16.600	0	0
10	PQ	1.027	0.000	-9.000	-5.800	0	0
11	PQ	1.044	0.000	-3.500	-1.800	0	0
12	PQ	1.051	0.000	-6.100	-1.600	0	0
13	PQ	1.044	0.000	-13.500	-5.800	0	0
14	PQ	1.014	0.000	-14.900	-5.000	0	0

Solution 3: Rescheduling the maintenance to a time when the system is less loaded.

If the consumer from bus 3 cannot be disconnected, another solution is to reschedule the maintenance during the night, valley hours, when the load is at its minimum. For this, the loads and generations from all buses are reduced to 60% from the previous values, with the bus 3 connected again, and the load flow is executed, with the new bus loads. The algorithm converges in 4 iterations, which means that with the new bus loads, the maintenance of line 1-2 is possible without jeopardizing the system's safety. The results are presented in Table 8.7.

Table 8.7 – Load flow results - the IEEE 14-bus system with line 1-2 disconnected and loads and generations reduced by 40%

bus no.	bus type	bus voltage magnitude [p.u.]	bus voltage angle [radians]	bus active power [MW]	bus reactive power [MVAr]	max. reactive power limit	min. reactive power limit
1	slack	1.060	0.000	146.360	4.996	0	0
2	PV	1.045	0.000	10.980	37.916	50	-40
3	PV	1.010	0.000	-56.520	-4.521	40	0
4	PQ	1.019	0.000	-28.680	2.340	0	0
5	PQ	1.017	0.000	-4.560	-0.960	0	0
6	PV	1.070	0.000	-6.720	4.563	24	-6
7	PQ	1.057	0.000	0.000	0.000	0	0
8	PQ	1.090	0.000	0.000	17.902	24	-6
9	PQ	1.044	0.000	-17.700	-9.960	0	0
10	PQ	1.043	0.000	-5.400	-3.480	0	0
11	PQ	1.054	0.000	-2.100	-1.080	0	0
12	PQ	1.059	0.000	-3.660	-0.960	0	0
13	PQ	1.054	0.000	-8.100	-3.480	0	0
14	PQ	1.036	0.000	-8.940	-3.000	0	0

References for C7 and C8:

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